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Bayesian Model of Collective Violence (Lynching)

(Collective) violence, once kindled, spreads through social learning among autonomous agents. In a sequence of events, autonomous agents perform a *Bayesian Update* on the available *State(s)* of the world after each violent event. Each event generates a set of informational externalities that (1) influence the beliefs of observers (about the legitimacy of violence), and (2) provides objective probabilities of sanctions. When update produces a state of the world in which likelihood of sanctioning is lower than the individual threshold for action, the observing agent engages in violence with a degree of certainty of getting away from authorities. When several autonomous agents engage in violence thinking permissive states of the world have been realized, we observe a global pattern of interaction that appears to be coordinated but as matter of fact manifests itself as an emergent property of multiple disjoint local interactions. An emergent pattern of behavior is self-sustaining even when some individuals change their initial behavior − unless a fundamental shift is introduced in the system.[1] Collective violence, hence, involves rational agents acting on the basis of direct observation or indirect information about other violent events processed in a Bayesian fashion.

To illustrates this point, consider lynching as a subset of collective vio-

[1]Anarchy as an ordering principle is an emergent property of international system in which states engage in self-help [Waltz, 2010]. It is self-sustaining in the sense that some states may decide disentangle themselves, but the system perpetrates itself (unless fundamental shift occurs [Wendt, 1999]).

lence, and a set of agents belonging to the class *lynchers* i.e. $\{l_1, l_2, l_3, l_4...l_i\}$. The propensity for lynching is determined by extremism distributed from less to more extreme as shown in Figure 3.[2]

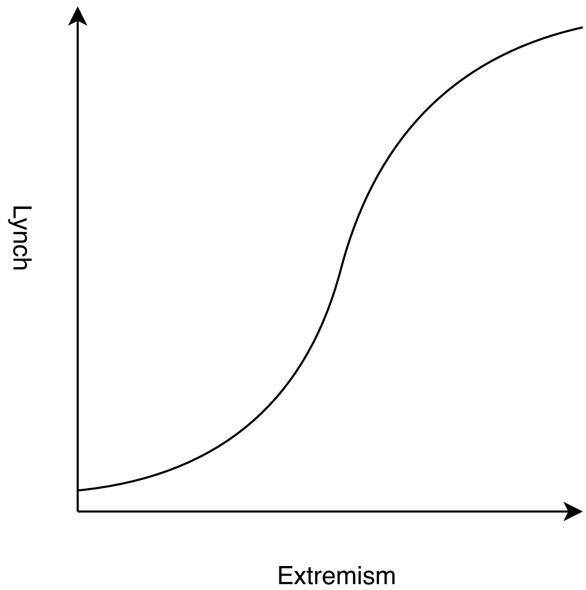

Figure 1: Hypothetical distribution of extremism and propensity to lynch.

The agents are faced with a binary choice: to lynch someone or not $\{L, \neg L\}$ in such a way that lynching is weakly preferred over not lynching meaning given an option to choose between these alternatives, agents will choose lynching more often if all else equal.

$$L \succeq \neg L$$

Consider extremism and lynching are directly related: more extremist agents are more prone to using violence and vice versa. Logically some agents will engage in lynching regardless of the consequences because they

[2]Different extremism distributions can be pulled from population data but consider this simple model for the time being. This assumption is based on Granovetter's threshold model [Granovetter, 1978].

will be willing to bear the cost and derive utility from the collectively optimal outcomes their actions would produce. Others will refrain from doing so because they would derive no utility (or alternatively bear costs) for their low extremism. The expected utility calculus for lynching someone can be stated as according to the general *expected utility theorem* [Von Neumann and Morgenstern, 1942]:

$$E(U) = (1 - p)U(B) + (p)U(B - C)$$

where E(U) refers to expected utility, p is the probability of getting sanctioned, (1 – p) is the perceived probability of getting away with lynching, B is perceived returns from lynching (either or both subjective and collective), and C is the perceived cost. This utility function describes two states: getting caught or getting away with lynching. When p = 1, the agent expects to get sanctioned certainty and, therefore, E(U) = U(B – C); that is, the expected utility of lynching is equal to the utility of the perceived returns of lynching minus the sanction issued by the authority. When p = 0, the agent expects to get away with certainty. Hence, the expected utility is equal to the utility of the perceived benefits E(U) = U(B). This equation suggests agents with binary weak preference for lynchings will engage in doing so if likelihood of getting away with lynching is higher or commensurate with the individual extremism.

The agent does not know the true state of the world however because there exists an infinitely many $\{s_1, s_2, s_3, s_4...s_i\}$. She performs a Bayesian update every time an event happens. The model assumes agents can observe perfect information.[3] The update reveals an approximate state of the world. Agents decides to engage in lynching if the revealed state correspond to her perceived state. Agent uses a simple Bayesian Rule to update her perceived state of the world at a given time t:

$$p(G|L) = \frac{p(L|G) \times p(G)}{p(L)}$$

where G stands for getting away with lynching and L stands for lynchings. The conditional probability $p(G|L)$ determines when an agent will take action or not. This probability is calculated based on aggregation of information generated by previous events. Agent's belief about the state of the world change and so does her likelihood of action each time a Bayesian update is performed. The likelihood of future lynchings at a given time t can then be calculated using:

$$p(L) = p(L|G) \times p(G) \quad + \quad p(L|\neg G) \times p(\neg G)$$

Figure 4. represents the decision calculus involved in lynching as a dynamic process. Let P_0 present the likelihood that *agent zero* (i.e. A_0) will engage in lynching. This agent is an extremist for argument's sake. A_i is any number of individual agents that are attentive to his action and

[3] This is a strong assumption (for example in the case of lynchings in India) it makes sense to assume so but given the scale and speed of digital connectivity.

the state of thew world S_i it reveals. A_i evaluates S_0 after A_0's action and makes a decision based on the aforementioned decision algorithm. If A_0 gets away with lynching, then A_1 decides to act and that state of the world S_1 becomes the prior knowledge for the next agent (as presented by the loop). A signal that arrests are proportional to lynchings will inhibit further lynchers, but the converse will encourage. A testable hypothesis can be inferred from this model:

Hypothesis: *The likelihood of future violent events increases of previous violent events go unpunished.*

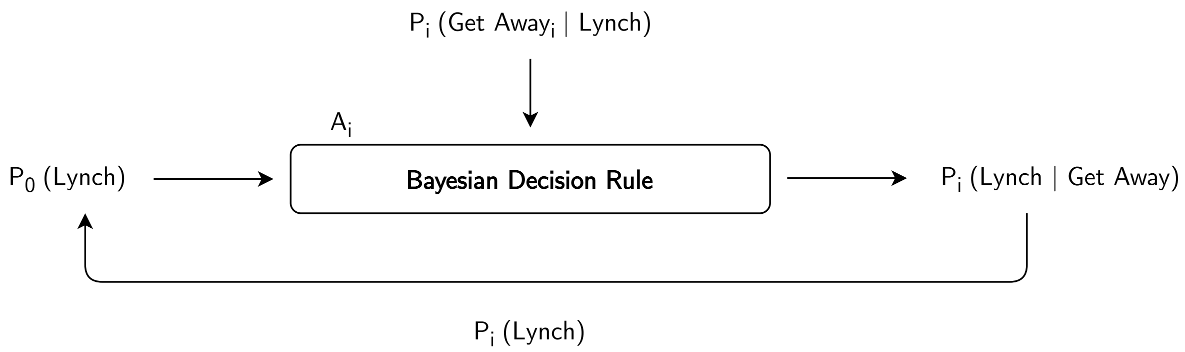

Figure 2: Process diagram of lynching conditioned on getting away.

To sum up, the aforementioned theory provides two insights besides outlining the possible strategic interaction between the violent agents and sanctioning authority. Collective violence can spring from a limited number of trigger incidents. Although the structural factors that generate such violence are important, the key to understanding variation in intra-unit (North-South lynching difference in USA) and inter-unit incidents of collective violence may be located in understanding how it spread, which is the second insight. Violence spreads through social learning, especially when decisions of the observing agent's are not simultaneous, but rationally calculated on the basis of information they receive from others, and distributed across time periods.

Model

Based on the mechanism defined above, consider a model of lynching in which a random agent's A_i *objective* probability of getting sanction after lynching some poor fellow is represented by $z_i \in [0,1]$. However, A_i's *subjective* probability (i.e. his belief about possible sanctions) is determined by whatever he knows about the true state of the world at a particular instance. At time t, which could be the time he engages in lynching, A_i's aforementioned belief will be simply determined by his risk perception at the end of previous instance *t-1*. Let that be $p_{i,t-1}$ i.e. subjective probability of sanctioning prior to time t.

The agent might have some rough idea where he stands but he is also aware that that the signal is noisy i.e. need further refinement. Two implications follow at this point: a extremist agent would engage in lynching based on that noisy signal due to his lower threshold for violence and disregard for sanctions, but a less extreme agent would act cautious and wait for further states of the world to reveal themselves and allow a better assessment of the situation. The level of extremism (or conversely tolerance for sanctions) may be determined by an assumed threshold distribution.

Let that noisy signal be $\theta_{it} \in [0,1]$ composed of the weighted sum of the observed state of the world denoted by s_i and l conducive factors denoted by $c_i^1, c_i^2, .., c_i^l$ of which k are observable. These factors may be situation specific ranging from limited resources [Sherif, 1966], relative deprivation [Gurr, 1970], population ratio [Blalock, 1967], remoteness of incident [Waldrep, 2002], absence of legitimate sanctioning structure [Pfeifer, 2011], to an ongoing communal strife [Balcells et al., 2016], group size [Lohmann, 1994], and permissive state apparatus [Adeel, 2019]. This signal can be presented as:

$$\theta_{it} = \delta_{it}(s_{it}) + \Sigma_j^l(w_{it}^j c_{it}^j)$$

where $\delta_{it} + \Sigma_j w_{it}^t = 1$ (for all i and t).

The real moving component that will affect a significant influence on the agent A_i is the state of the world that is revealed as a proportion of agents getting away with lynching. The equation above can be modified as:

$$\theta_{it} = \delta_{it}(G_{it}/L_{it}) + \Sigma_j^l(w_{it}^j c_{it}^j) \tag{1}$$

Whereas G_{it} represents no sanctions for previous lynchings L_{it} that A_i observes at the start of t. These factors vary with agents and time as indicated through the inclusion of timestep t in the subscripts.

As refined signals will reveal new information at each iteration, A_i (assumed he is cautious) will gradually become confident in how θ_{it} or subjective probability of sanctions corresponds to the objective or true probability z_i. A_i will then form a posterior belief in the likelihood of sanctions given lynching based on this the revealed state of the world at time t and prior information that he holds before time t. That again is a weighted average of these two components:

$$p_{it} = \alpha_{it}(\theta_{it}) + (1 - \alpha_{it})(p_{i,t-1})$$

where $\alpha_{it} + (1 - \alpha_{it}) = 1$ (for all i and t).

Substituting the value of θ_{it} from Eq. 1, we have

$$p_{it} = \alpha_{it}[\delta_{it}(G_{it}/L_{it}) + \Sigma_j^l(w_{it}^j c_{it}^j)] + (1 - \alpha_{it})(p_{i,t-1})$$

Driving α_{it} inside, we get

$$p_{it} = \alpha_{it}\delta_{it}(G_{it}/L_{it}) + \alpha_{it}\Sigma_j^l(w_{it}^j c_{it}^j) + (1 - \alpha_{it})(p_{i,t-1})$$

$$p_{it} = \alpha_{it}\delta_{it}(G_{it}/L_{it}) + \alpha_{it}(\Sigma_{j=1}^k(w_{it}^j c_{it}^j) + \Sigma_{j=k+1}^l(w_{it}^j c_{it}^j)) + (1 - \alpha_{it})(p_{i,t-1})$$

$$p_{it} = \beta_{0t} + \beta_{1t}(G_{it}/L_{it}) + \beta_{2t}(p_{i,t-1}) + \Sigma_{m=1}^k \beta_{(m+2)t} c_{it}^m$$

where $\beta_{0t} = \alpha_{it}\Sigma_{j=k+1}^l(w_{it}^j c_{it}^j)$ (or the combined effect of un-observed factors), $\beta_{1t} = \alpha_{it}\delta_{it}$, $\beta_{2t} = 1 - \alpha_{it}$, and $\beta_{(m+2)t} = \alpha_{it} w_{it}^m$ (for m=1..k).

Assuming we want to find the average effect of the β's over all time steps t, we can drop the timestamps from the β subscripts as below.

$$p_{it} = \beta_0 + \beta_1(G_{it}/L_{it}) + +\beta_2(p_{i,t-1}) + \Sigma_{m=1}^{k}\beta_{m+2}c_{it}^{m} + \epsilon_{it} \tag{2}$$

where $\epsilon_{it} = (\beta_{0t} - \beta_0) + (\beta_{1t} - \beta_1)(G_{it}/L_{it}) + \Sigma_{m=1}^{k}(\beta_{(m+2)t} - \beta_{m+2})c_{it}^{m})$

Now, the coefficients based on regression relation generated by Eq. 2 can be estimated using ordinary least squares (OLS).

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